Absence of two-flavor color superconductivity in compact stars

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Abstract

The simplest pattern of color superconductivity involves BCS pairing between up and down quarks. We argue that this "2SC" phase, and variants of it, will not arise within a compact star. A macroscopic volume of quark matter must be electrically neutral and must be a color singlet. Satisfying these requirements imposes a significant free energy cost on the 2SC phase, but not on color-flavor locked (CFL) quark matter, in which up, down and strange quarks all pair. As a function of increasing density, therefore, one may see a single phase transition from hadronic matter directly to CFL quark matter. Alternatively, there may be an intervening phase in which the different flavors self-pair, or pair with each other in a non-BCS pattern, such as in a crystalline color superconductor.

1 Introduction

We are beginning to learn some interesting things about matter at very high densities. For experimental information, we have to rely on observations of compact stars, which consist of matter at nuclear density at the surface, increasing to an unknown maximum density at the core. So far these observations have not put strong constraints on the behavior of matter beyond nuclear density. Theoretically, several different possible phases have been conjectured for hyperdense matter of the kind we would expect to find in compact stars. Some of the most interesting possibilities are the color-superconducting phases expected if the density is high enough to transform hadrons into quark matter [1, 2, 3, 4, 5, 6]. Color superconductivity occurs because QCD predicts an attractive force between quarks which are antisymmetric in color, so we expect quarks near their Fermi surfaces to form Cooper pairs, which condense, breaking the color gauge symmetry. One pairing pattern, known to be favored at sufficiently high density, is the color-flavor locked (CFL) phase in which up-down, down-strange, and up-strange Cooper pairs all form, allowing quarks of all three colors and all three flavors to pair [5]. At lower densities, where the strange quark mass disfavors pairing of the strange with the light quarks, previous work has suggested that the favored phase is a form of color superconductivity in which up and down quarks of two of the three colors participate in pairing. This "2SC" phase leaves the third color and strange quarks unpaired.

In this paper we investigate what forms of superconducting quark matter are most likely to occur in compact stars, taking into account the constraints that must be imposed for a stable bulk phase. These are:

- Electromagnetic neutrality. Bulk matter must be neutral with respect to any Abelian gauge charge (broken or unbroken). If it were not, the energy would be nonextensive, growing faster than the volume as the size of the sample increases.

- Color neutrality. A macroscopic chunk of quark matter must be a color singlet. Color neutrality, the equality in the number density of red, green and blue color charges, is a prerequisite for this. Indeed, the authors of Ref. [7] have shown that so long as a macroscopic chunk of color superconductor is color neutral, carrying out the projection which imposes color singlet-ness has negligible effect on the free energy of the state. This result is analogous to the familiar fact from ordinary superconductivity that the projection which turns a BCS state, wherein particle number is formally indefinite, into a state with definite but very large particle number has no significant effect. Here, "macroscopic" means that the chunk of quark matter must be much larger than the inverse of the gap Δ [7], which is of order 10 to 100 MeV in a color superconductor [3, 4, 5, 6]. We are interested in color superconducting regions of order kilometers in size. Thus, to very good approximation all we need to worry about is color neutrality. Color singlet-ness follows without paying any further free energy price.

We shall see that when we impose these constraints, the free energy of the 2SC phase becomes large enough that it is unlikely to be found in nature. The CFL phase, by contrast, satisfies the neutrality constraints automatically [8].

Early work on the 2SC phase ignored issues of color and electromagnetic neutrality, focussing instead on foundational issues like the size of the pairing gap. Calculations were performed in various toy models in which QCD with only two flavors of quarks was analyzed with the assumption of equal chemical potentials and equal number densities for up and down quarks. Subsequently, many authors have assumed that there is some range of values of the strange quark mass for which up-strange and down-strange pairing can be neglected, and so a stable bulk 2SC phase would occur. We argue here that this is not the case. If the strange quark mass precludes up-strange and down-strange pairing, then up-down pairing (ie the 2SC phase) is also precluded. Other authors have looked at the implications of maintaining electric and color neutrality in the 2SC phase [9], but the very existence of stable bulk 2SC quark matter has not previously been questioned.

In this paper we shall consider matter containing up (u), down (d) and strange (s) quarks, but no heavier quarks. We will take the u and d to be massless, and the strange quark to have an effective ("constituent") mass M_s, which may depend on the density and phase, and is larger than its current mass $m_s \approx 120$ MeV. We shall assume that the quark number chemical potential μ is large compared to M_s, and work only to lowest nontrivial order in M_s^2/μ^2. This simplifies our analysis considerably, but we should remark that it is unlikely that μ is much larger than 500 MeV even in the center of a compact star, meaning that M_s^2/μ^2 may not be very small.

2 Enforcing neutrality

We shall limit our discussion to quark matter in compact stars older than a few minutes. This justifies our working at zero temperature, since the temperature is by then well below the expected value of all gaps and quasiparticle masses. It also means that there has been plenty of time to come to equilibrium under the weak interactions,[1] so flavor symmetries are broken. It also means that the star is transparent to neutrinos, which leave the system, so lepton number is not conserved. The relevant symmetries are therefore just the color and electromagnetic gauge symmetries and the global symmetry related to baryon number conservation:

$$[SU(3)_{\text{color}}] \times [U(1)_Q] \times U(1)_B \ . \tag{2.1}$$

[1]Other authors have considered color superconductivity in contexts in which the weak interactions are not in equilibrium, as would for example be appropriate if it were somehow possible to realize color superconductivity in a heavy ion collision [10].

In any color superconducting phase, this symmetry is broken down to some subgroup by condensation of quark pairs.

The baryon number of an isolated macroscopic body is fixed, but since baryon number is not a gauge symmetry, nothing prevents matter from having a constant nonzero baryon density in the infinite volume limit. There is no requirement for "baryon-number neutrality", and we can treat the chemical potential for baryon number, 3μ, as a free parameter. A color superconductor is a BCS state in which baryon number is spontaneously broken by virtue of a condensate wherein $\langle qq\rangle \neq 0$. A macroscopic color superconductor may therefore seem to have ill-defined baryon number, but just as in an ordinary superconductor or superfluid, and as shown explicitly in Ref. [7], projecting the BCS state onto a state of fixed large baryon number has no significant effect.

Because we are only concerned with enforcing color neutrality, as opposed to color singlet-ness, we need only consider the the $U(1)_3 \times U(1)_8$ subgroup of the color gauge symmetry generated by the Cartan subalgebra $T_3 = \mathrm{diag}(\frac{1}{2}, -\frac{1}{2}, 0)$ and $T_8 = \mathrm{diag}(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3})$ in color space, where we choose to label the colors as (r, g, b). We introduce chemical (color-electrostatic) potentials μ_3 and μ_8 coupled to the color charges T_3 and T_8. Enforcing color neutrality means choosing μ_3 and μ_8 such that the T_3 and T_8 densities vanish. Enforcing T_3 and T_8 neutrality suffices to enforce equality in the number of red, green and blue quarks.

Electromagnetism is generated by $Q = \mathrm{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ in flavor space, where we order the flavors as (u, d, s). We will focus on the negative charge $Q_e = -Q$, coupling it to the (negative) electrostatic potential μ_e. ($\mu_e > 0$ corresponds to a density of electrons; $\mu_e < 0$ to positrons.) Enforcing electric neutrality means choosing μ_e so that Q_e vanishes.

In a color superconducting phase, there is an expectation value for some diquark operator. Since a diquark cannot be electrically neutral and cannot be color neutral, some subgroup of $U(1)_3 \times U(1)_8 \times U(1)_Q$ is spontaneously broken. As we shall see, however, there is typically at least one linear combination of T_3, T_8 and Q with respect to which the condensate is neutral, meaning that there is at least one $U(1)$ subgroup of $U(1)_3 \times U(1)_8 \times U(1)_Q$ which the condensate leaves unbroken. We now discuss unbroken and broken gauged charges in turn.

Unbroken gauged charges. Any uniform phase (we will not consider mixed phases, although they may be important in some contexts), must be neutral under unbroken gauged charges, to avoid the infrared-divergent energy cost of long-range electric fields. The corresponding chemical (ie electrostatic) potential is forced to the value of μ_Q that solves

$$Q = \frac{\partial \Omega}{\partial \mu_Q} = 0. \tag{2.2}$$

Choosing this value of μ_Q changes the contribution of any particles with Q-charge to the free energy of the system. The ensuing free energy cost is proportional to the volume, though, whereas the free energy cost of the electric fields that would arise if neutrality were not enforced grows with system size faster than the volume.

A macroscopic but finite chunk of matter, that has $\mu_Q \neq 0$ within it chosen so that it is neutral, has a Q-electric field (*i.e.* a gradient in μ_Q) across its surface. This Q-electric field corresponds to a layer of polarization. (For analogous surface phenomena occuring at the interface between nuclear and quark matter, see Ref. [11].) We will not consider surfaces in this paper. For our purposes, it suffices to choose μ_Q to enforce Q-neutrality, thus obtaining matter which can sensibly be analyzed in the infinite volume limit.

Broken gauged charges. The argument requiring that any uniform phase be neutral applies to broken symmetries too. If a finite-sized sample has a net charge then there will be electric fields outside the sample that grow in strength with the size of the sample. This means that the sample must be neutral to have a good large-volume limit.

Thus, we can treat T_3, T_8 and Q on the same footing, regardless of which linear combinations are broken and which are unbroken.

3 Free energy comparisons

In this section, we compute the free energy of noninteracting (and thus unpaired) quark matter, CFL quark matter, 2SC quark matter, and several variants of 2SC quark matter. We work to order $(M_s/\mu)^4$ and $(\Delta/\mu)^2$. We shall see that the free energies of these different phases must be compared in a regime where $\Delta \sim M_s^2/\mu$, meaning that an expansion of this sort is formally consistent. The analysis would be considerably more difficult were we to attempt to work to higher order in M_s, but this will not be necessary to make the qualitative points we wish to make. Quantitatively, we are interested in $\mu \sim 500$ MeV and it is likely that $120 < M_s < 500$ MeV while $\Delta \sim 10$ to 100 MeV.

We shall work in unitary gauge, where the color-direction of the quark pairing is chosen to be position-independent. It is always possible to make gauge transformations which make the color-direction of the quarks vary with position and transfer some color into the off-diagonal glue fields. This would change nothing, although it would make the analysis less transparent.

In our analysis, we do not include any contribution to the free energy from the nonabelian gauge bosons. In principle, these can carry charge and so one may worry that they contribute to maintaining (or upsetting) color neutrality. In unitary gauge, the color current carried by gluons arises from the gauge-connection part of the covariant derivative of the gauge field, $J^\nu_{\rm glue} = g[A_\mu, F^{\mu\nu}]$. In order for the charge $J^0_{\rm glue}$ to be nonzero, there would have to be a nonzero color electric field F^{i0}, which is impossible in a conducting or broken (superconducting) phase.

3.1 Unpaired quark matter

We begin with noninteracting quark matter. If there really were no interactions, and therefore global symmetries only, there would be no reason for the quark matter to be neutral in any sense. So, we imagine turning on arbitrarily small gauge couplings, as this motivates the requirement that we impose electric and color neutrality and satisfy weak equilibrium, and ask what chemical potentials we must introduce in order to achieve neutrality. Given that quark masses are independent of color, we simply set $\mu_3 = \mu_8 = 0$ and obtain quark matter in which the up, down and strange quarks are all separately color neutral. Imposing electrical neutrality is (a little) more challenging. Weak equilibrium relates the chemical potentials for the three flavors to just μ and μ_e:

$$\begin{aligned} \mu_u &= \mu - \tfrac{2}{3}\mu_e \\ \mu_d &= \mu + \tfrac{1}{3}\mu_e \\ \mu_s &= \mu + \tfrac{1}{3}\mu_e \ , \end{aligned} \tag{3.1}$$

independent of color because $\mu_3 = \mu_8 = 0$. The free energy is minimized by filling Fermi seas for each quark flavor, up to Fermi momenta given by:

$$\begin{aligned} p_F^u &= \mu_u \\ p_F^d &= \mu_d \\ p_F^s &= \sqrt{\mu_s^2 - M_s^2} \ , \end{aligned} \tag{3.2}$$

since the energy of a strange quark with momentum p_F is $\sqrt{p_F^2 + M_s^2}$. The free energy of unpaired quark matter is then given by

$$\begin{aligned} \Omega_{\text{unpaired}}(\mu, \mu_e, M_s) &= \frac{3}{\pi^2}\int_0^{p_F^u}(p-\mu_u)p^2dp + \frac{3}{\pi^2}\int_0^{p_F^d}(p-\mu_d)p^2dp \\ &+ \frac{3}{\pi^2}\int_0^{p_F^s}(\sqrt{p^2+M_s^2}-\mu_s)p^2dp + \frac{1}{\pi^2}\int_0^{\mu_e}(p-\mu_e)p^2dp \ . \end{aligned} \tag{3.3}$$

The last term, which is just $-\mu_e^4/12\pi^2$, is the free energy of the electrons which are present because $\mu_e > 0$. We now fix μ_e by requiring electrical nautrality

$$\frac{\partial \Omega_{\text{unpaired}}}{\partial \mu_e} = 0 \ . \tag{3.4}$$

To lowest nontrivial order in M_s, this yields

$$\mu_e = \frac{M_s^2}{4\mu} \ , \tag{3.5}$$

meaning that in electrically neutral noninteracting quark matter the Fermi momenta are given by

$$p_F^d = \mu + \frac{M_s^2}{12\mu} = p_F^u + \frac{M_s^2}{4\mu}$$

$$
\begin{aligned}
p_F^u &= \mu - \frac{M_s^2}{6\mu} \\
p_F^s &= \mu - \frac{5M_s^2}{12\mu} = p_F^u - \frac{M_s^2}{4\mu}\ ,
\end{aligned}
\tag{3.6}
$$

to lowest order. Once (3.4) is satisfied, μ_e does not occur linearly in Ω. This means that to see the first effects of μ_e, we need to work to order $\mu_e^2 \sim M_s^4$. To this order, upon substituting (3.5) into (3.3) we find

$$
\Omega_{\rm unpaired}^{\rm neutral} = -\frac{3\mu^4}{4\pi^2} + \frac{3M_s^2\mu^2}{4\pi^2} - \frac{7 - 12\log(M_s/2\mu)}{32\pi^2} M_s^4\ . \tag{3.7}
$$

Several points are worth noting before we proceed:

- The electronic contribution to Ω is of order $\mu_e^4 \sim M_s^8$, and therefore does not appear in (3.7). We therefore neglect it from the beginning in our analyses of paired phases below. But, in any of the phases we discuss below in which $\mu_e \neq 0$, electrons are present and affect Ω at order M_s^8.

- Below, the only effect of interactions which we shall include are those due to BCS pairing, resulting in gaps Δ for some or all of the quarks. Interactions of course result in other changes to the free energy. For example, the perturbative (in the QCD coupling constant) corrections to the free energy are well-studied [12]. The leading effect is a change in the coefficient of the μ^4 term in the free energy. These effects are the same for all the phases of quark matter we consider, and thus cancel in the comparisons we shall make. (More precisely, any differences between the free energies of the different color superconducting phases introduced by perturbative QCD interactions are perturbative corrections to the $\Delta^2\mu^2$ effects we consider explicitly.) These effects would matter only if we compared the free energy of any of the quark matter phases we analyze to that of hadronic matter. Similarly, we leave the bag constant out since it also only matters in comparisons between hadronic and quark matter. We shall not attempt such comparisons.

- Note that the effect of the strange quark mass, combined with the requirement of electric neutrality, is to push p_F^d up and p_F^s down relative to p_F^u. It is clear that, at some point, this will cause the CFL phase in which all flavors pair with each other to be disfavored relative to unpaired quark matter. The question we seek to answer is whether at that point pairing of up with down remains possible.

3.2 Color-flavor locked quark matter

In color-flavor locked quark matter [5], quarks form a condensate in which

$$
\langle q_a^\alpha C\gamma_5 q_b^\beta \rangle \sim \Delta \left(\epsilon^{\alpha\beta 1}\epsilon_{ab1} + \epsilon^{\alpha\beta 2}\epsilon_{ab2} + \epsilon^{\alpha\beta 3}\epsilon_{ab3} \right)\ , \tag{3.8}
$$

where α, β are color indices (r, g, b) and a, b are flavor indices (u, d, s). This form follows from the QCD interaction, which is attractive in the color-antisymmetric channel, and from requiring that rotational invariance be preserved, which leads to a spin-antisymmetric state. The flavor structure is then forced to be antisymmetric.[2] Each of the three terms in (3.8) satisfy these antisymmetry requirements; summing the three of them allows all nine quarks to pair, maximizing the pairing energy. In this state, rd and gu quarks pair yielding two quasiparticles with gap Δ, rs and bu quarks pair yielding two quasiparticles with gap Δ, bd and gs quarks pair yielding two quasiparticles with gap Δ, and ru, gd and bs quarks pair yielding two quasiparticles with gap Δ and one with gap 2Δ. The $U(1)_3 \times U(1)_8 \times U(1)_Q$ symmetry is broken to $U(1)_{\tilde{Q}}$, where $\tilde{Q} = Q - T_3 - \frac{1}{2}T_8$ is the generator of the unbroken symmetry.

To order Δ^2, the free energy of the CFL phase can be described quite simply. One begins with the free energy of a (fictional) state of unpaired quark matter in which all quarks which are "going to pair" have a common Fermi momentum p_F^{common}, with p_F^{common} chosen to minimize the free energy of this fictional unpaired state. The binding energy of the diquark condensate is included by subtracting $\Delta^2\mu^2/4\pi^2$ for every quasiparticle with gap Δ (assuming that μ_e, μ_3, μ_8 are all of order M_s^2 or smaller, which will turn out to be true for all the phases we study). Thus,

$$\Omega_{\text{CFL}} = \frac{1}{\pi^2}\sum_{i=1}^{9}\int_0^{p_F^{\text{common}}}\left(\sqrt{p^2+M_i^2}-\mu_i\right)p^2dp - \frac{3}{\pi^2}\Delta^2\mu^2 \ , \tag{3.9}$$

where the sum runs over all nine quarks, where $M_i = 0$ for the up and down quarks and $M_i = M_s$ for the strange quarks, and where $\mu_i = \mu - Q\mu_e + T_3\mu_3 + T_8\mu_8$ is determined for each quark by its electric and color charges. This form for the free energy of a state with BCS pairing between different species goes back to the work of Clogston and Chandrasekhar [14], and was derived in the CFL context in Refs. [8, 11]. Note that it is only valid to order Δ^2 and M_s^4, and that it follows from substituting back into a more general expression the value of Δ that solves the gap equation, in other words it has already been minimized with respect to Δ [15].

We emphasize that, to this order, the nature of the interaction which generates Δ does not matter. The free energy is given by this prescription regardless of whether the pairing is due to a point-like four-fermi interaction, as in NJL models, or due to the exchange of a gluon, as in QCD at asymptotically high energies [15]. Of course, the strength and form of the interaction determine the value of Δ, which we shall keep as a free parameter.

For massless quarks, evaluating p_F^{common} yields simply the average of the chemical potentials of the quarks which pair. The lowest order effect of M_s on p_F^{common} is to weight the strange quarks in this average as if their chemical potential were depressed

[2]We neglect the additional condensate which is symmetric in color and flavor, because although it must be nonzero it is much smaller than the antisymmetric condensate (3.8) [5], and because its presence does not modify the symmetries of CFL quark matter [13].

by $M_s^2/2\mu$. The average of μ_e, μ_3 and μ_8 are each zero for the nine quarks which pair in the CFL phase, meaning that $p_F^{\rm common}$ is independent of μ_e, μ_3 and μ_8 and is given, to lowest order, by

$$p_F^{\rm common} = \mu - \frac{M_s^2}{6\mu} \ . \tag{3.10}$$

Knowing $p_F^{\rm common}$ to this order is sufficient to obtain $\Omega_{\rm CFL}$ to order M_s^4. Note that the quark number density in the CFL phase is *not* simply proportional to $(p_F^{\rm common})^3$: $\partial\Omega_{\rm CFL}/\partial\mu$ receives a contribution also from the $\Delta^2\mu^2$ term. And, of course, in a paired state there is no sharp Fermi surface anyway as the single particle density of states is smeared out by of order Δ. Thus, the fictional unpaired quark matter used in the construction of $\Omega_{\rm CFL}$ is fictional in two senses: (i) because all Fermi momenta are given by $p_F^{\rm common}$, this is *not* the unpaired quark matter of Section 3.1 that is found at a given μ, μ_e, μ_3, μ_8 in the absence of interaction; (ii) the CFL state has no Fermi momentum and the parameter $p_F^{\rm common}$ that arises in its description does not fully specify the quark number density in the CFL state.

To the order at which we are working,

$$\frac{\partial\Omega_{\rm CFL}}{\partial\mu_e} = \frac{\partial\Omega_{\rm CFL}}{\partial\mu_3} = \frac{\partial\Omega_{\rm CFL}}{\partial\mu_8} = 0 \ , \tag{3.11}$$

meaning that we find electric and color neutrality with $\mu_e = \mu_3 = \mu_8 = 0$. In fact, to this order it appears that we can set these chemical potentials to any value we like without upsetting neutrality, but this need not hold at higher order as we discuss below. To order M_s^4, the free energy for electric and color neutral CFL quark matter is given by [11]

$$\begin{aligned}
\Omega_{\rm CFL}^{\rm neutral} &= -\frac{3\mu^4}{4\pi^2} + \frac{3M_s^2\mu^2}{4\pi^2} - \frac{1-12\log(M_s/2\mu)}{32\pi^2}M_s^4 - \frac{3\Delta^2\mu^2}{\pi^2} \\
&= \Omega_{\rm unpaired}^{\rm neutral} + \frac{3M_s^4 - 48\Delta^2\mu^2}{16\pi^2} \ .
\end{aligned} \tag{3.12}$$

We conclude that CFL quark matter is favored over unpaired quark matter as long as the interactions are strong enough to generate a gap in the CFL phase satisfying

$$\Delta > \frac{M_s^2}{4\mu} \ . \tag{3.13}$$

Since Δ is of order M_s^2 in this criterion, our strategy of evaluating Ω to order M_s^4 and to order Δ^2 is consistent.

We have neglected the fact that when $M_s \neq 0$, the gaps in the CFL phase are M_s-dependent and, furthermore, are non-degenerate [13, 16]. For example, the gap for the quasiparticles obtained upon pairing the rd and gu quarks differs from that obtained upon pairing the rs and bu quarks. The leading M_s dependence of the latter is $\Delta(M_s) \sim \Delta(0)(1 - cM_s^2/\mu^2)$, for some coefficient c [17]. These effects are therefore of order $\Delta(0)^2 M_s^2$ in the free energy and it is consistent to neglect them. Although their neglect is consistent, they are important because they introduce μ_e, μ_3 and μ_8 dependence into the free energy. That is, at a high enough order that the differences among gaps matters, the fact that different Cooper pairs in the CFL phase have different Q, T_3 and T_8 charges matters. (At this order, the expression (3.9) is itself incomplete.) It is worth noting that the M_s-dependent differences among gaps cannot introduce any dependence on the $\tilde{Q}$ chemical potential because, by the definition of $\tilde{Q}$, each Cooper pair in the CFL phase has $\tilde{Q}=0$. (For example, rd with $\tilde{Q}=-1$ pairs only with gu with $\tilde{Q}=+1$.) The CFL phase is $\tilde{Q}$-neutral, and in fact is a $\tilde{Q}$-insulator [8], even at higher order in M_s. On the other hand, CFL Cooper pairs do have T_3 and T_8 charges. The total T_3 and T_8 charge of the condensate vanishes only to the extent that all the gaps are the same. This means that imposing neutrality with respect to the broken gauge charges of the CFL phase will be less trivial at higher order than it is at the order we are working. This warrants investigation.

We have also neglected the possibility that when $M_s \neq 0$, the left-handed CFL condensate and the right-handed CFL condensate may rotate relative to one another in flavor space [18, 19]. By neglecting this kaon condensation, we have neglected an effect which, if it occurs, lowers the free energy of the CFL phase at order M_s^4 [18] meaning that in this instance our neglect is not consistent. If we take the quantitative results for the correction to $\Omega_{\rm CFL}$ due to maximal kaon condensation given in Ref. [18], we find that the 4 in the denominator of (3.13) increases but remains smaller than 5. We shall see below that in the comparison between CFL and 2SC, there is a much larger uncertainty to be concerned with.

3.3 2SC quark matter

In 2SC quark matter, quarks form a condensate in which

$$\langle q_a^\alpha C\gamma_5 q_b^\beta\rangle \sim \Delta\epsilon^{\alpha\beta 3}\epsilon_{ab3}\ . \tag{3.14}$$

In this state, rd and gu quarks pair yielding two quasiparticles with gap Δ, and ru and gd quarks pair, yielding two quasiparticles with gap Δ. Five quarks are left unpaired. The $U(1)_3$ symmetry (and indeed a color $SU(2)$ symmetry of which it is a subgroup) is left unbroken, as is $U(1)_{\tilde{Q}}$, where $\tilde{Q}=Q-\frac{1}{2}T_8$.

To the order we are working, the free energy of the 2SC phase is

$$\begin{aligned}\Omega_{\rm 2SC} &= \frac{1}{\pi^2}\sum_{i=1}^{4}\int_0^{p_F^{\rm common}}(p-\mu_i)p^2dp\\ &+\frac{1}{\pi^2}\sum_{i=5}^{9}\int_0^{\sqrt{\mu_i^2-M_i^2}}\left(\sqrt{p^2+M_i^2}-\mu_i\right)p^2dp\\ &-\frac{1}{\pi^2}\Delta^2\mu^2\ ,\end{aligned}\tag{3.15}$$

where the first four quarks are those that pair while the last five are those that don't. This time, $p_F^{\rm common}$ is just the average of the chemical potentials of the quarks which pair:

$$p_F^{\rm common} = \mu - \tfrac{1}{6}\mu_e + \tfrac{1}{3}\mu_8 \ . \tag{3.16}$$

To the order at which we are working,

$$\begin{aligned}
\frac{\partial\Omega_{\rm 2SC}}{\partial\mu_e} &= \frac{M_s^2\mu - 2\mu_e\mu^2}{2\pi^2} \ , \\
\frac{\partial\Omega_{\rm 2SC}}{\partial\mu_3} &= -\frac{\mu_3\mu^2}{2\pi^2} \ , \\
\frac{\partial\Omega_{\rm 2SC}}{\partial\mu_8} &= -\frac{2\mu_8\mu^2}{\pi^2} \ ,
\end{aligned} \tag{3.17}$$

meaning that we find electric and color neutrality with $\mu_3 = \mu_8 = 0$ and $\mu_e = M_s^2/2\mu$. Note that Q is a linear combination of the unbroken charge $\tilde{Q}$ and the orthogonal broken charge $X \equiv T_8 - \frac{1}{2}Q$. Thus, $\mu_8 = 0$ and $\mu_e = M_s^2/2\mu$ corresponds, when written in terms of the charges which are natural to use to describe the 2SC phase, to $\mu_{\tilde{Q}} = M_s^2/2\mu$ and $\mu_X = -M_s^2/4\mu$. The 2SC phase contains ungapped $\tilde{Q}$-carrying modes, and is therefore a $\tilde{Q}$-conductor.

To order M_s^4, the free energy for electric and color neutral 2SC quark matter is given by

$$\begin{aligned}
\Omega_{\rm 2SC}^{\rm neutral} &= -\frac{3\mu^4}{4\pi^2} + \frac{3M_s^2\mu^2}{4\pi^2} - \frac{5-12\log(M_s/2\mu)}{32\pi^2}M_s^4 - \frac{\Delta^2\mu^2}{\pi^2} \\
&= \Omega_{\rm unpaired}^{\rm neutral} + \frac{M_s^4 - 16\Delta^2\mu^2}{16\pi^2} \ .
\end{aligned} \tag{3.18}$$

3.4 Variants of 2SC

There are two variants of the 2SC phase which we must consider, before we make the comparison between 2SC and CFL. In the 2SC phase above, it is the u and d quarks that pair. Given (3.6), it seems as likely for u and s to pair. And, given (3.6), why not make both u-s and u-d pairs, while leaving d and s unpaired?

3.4.1 2SCus quark matter

In 2SCus quark matter, quarks form a condensate in which

$$\langle q_a^\alpha C\gamma_5 q_b^\beta\rangle \sim \Delta\epsilon^{\alpha\beta 2}\epsilon_{ab2} \ . \tag{3.19}$$

In this state, rs and bu quarks pair yielding two quasiparticles with gap Δ, and ru and bs quarks pair, yielding two quasiparticles with gap Δ. Five quarks are left unpaired. The color $U(1)$ symmetry generated by $\mathrm{diag}(\frac{1}{2}, 0, -\frac{1}{2}) = \frac{1}{2}T_3 + \frac{3}{4}T_8$ (and indeed a color $SU(2)$ symmetry of which this is a subgroup) is left unbroken, as is $U(1)_{\tilde{Q}}$, where $\tilde{Q} = Q - T_3 - \frac{1}{2}T_8$.

To the order we are working, the free energy of the 2SCus phase is

$$\begin{aligned}
\Omega_{\rm 2SCus} &= \frac{1}{\pi^2}\sum_{i=1}^{4}\int_0^{p_F^{\rm common}} (\sqrt{p^2+M_i^2}-\mu_i)p^2dp \\
&+ \frac{1}{\pi^2}\sum_{i=5}^{9}\int_0^{\sqrt{\mu_i^2-M_i^2}} (\sqrt{p^2+M_i^2}-\mu_i)p^2dp - \frac{1}{\pi^2}\Delta^2\mu^2 \ ,
\end{aligned} \tag{3.20}$$

where the first four quarks are those that pair while the last five are those that don't. This time,

$$p_F^{\rm common} = \mu - \tfrac{1}{6}\mu_e + \tfrac{1}{3}\mu_8 - \frac{M_s^2}{4\mu} \ . \tag{3.21}$$

To the order at which we are working,

$$\begin{aligned}
\frac{\partial\Omega_{\rm 2SCus}}{\partial\mu_e} &= -\frac{\mu_e\mu^2}{\pi^2} \\
\frac{\partial\Omega_{\rm 2SCus}}{\partial\mu_3} &= -\frac{\mu_3\mu^2}{2\pi^2} \\
\frac{\partial\Omega_{\rm 2SCus}}{\partial\mu_8} &= -\frac{2\mu_8\mu^2}{\pi^2} \ ,
\end{aligned} \tag{3.22}$$

meaning that we find electric and color neutrality with $\mu_3 = \mu_8 = \mu_e = 0$. (At higher order, all would become nonzero.) The 2SCus phase is a $\tilde{Q}$-conductor.

To order M_s^4, the free energy for electric and color neutral 2SCus quark matter is identical to that for electric and color neutral 2SC quark matter, given in (3.18).

3.4.2 Double-2SC quark matter

If we wish to write a condensate in which u pairs with d and u pairs with s, and wish to maintain antisymmetry in color, which the QCD interaction favors, the only choices are linear combinations of the 2SC and 2SCus condensates, or color-rotations thereof. For simplicity, we shall only consider the form

$$\langle q_a^\alpha C\gamma_5 q_b^\beta\rangle \sim \Delta\left(\epsilon^{\alpha\beta 2}\epsilon_{ab2} + \epsilon^{\alpha\beta 3}\epsilon_{ab3}\right) \ . \tag{3.23}$$

In this state, which we shall call the "Double-2SC" phase, rd and gu quarks pair yielding two quasiparticles with gap Δ, rs and bu quarks pair yielding two quasiparticles with gap Δ, and ru, gd and bs quarks pair, yielding two quasiparticles with gap $\sqrt{2}\Delta$ and one gapless quasiparticle. The gs and bd quarks are left unpaired. Only the $U(1)$ symmetry generated by $\tilde{Q} = Q - T_3 - \frac{1}{2}T_8$ is left unbroken.

To the order we are working, the free energy of the Double-2SC phase is

$$\Omega_{\rm Double-2SC} = \frac{1}{\pi^2}\sum_{i=1}^{7}\int_0^{p_F^{\rm common}}\left(\sqrt{p^2+M_i^2}-\mu_i\right)p^2dp + \frac{1}{\pi^2}\sum_{i=8}^{9}\int_0^{\sqrt{\mu_i^2-M_i^2}}\left(\sqrt{p^2+M_i^2}-\mu_i\right)p^2dp - \frac{2}{\pi^2}\Delta^2\mu^2\ , \quad (3.24)$$

where the first seven quarks are those that pair while the last two are those that don't. This time,

$$p_F^{\rm common} = \mu - \frac{2\mu_e}{21} + \frac{\mu_3}{14} + \frac{\mu_8}{21} - \frac{M_s^2}{7\mu}\ . \quad (3.25)$$

To the order at which we are working, electric and color neutrality are obtained by setting $\mu_3 = \mu_e - M_s^2/2\mu$ and $\mu_8 = \mu_e/2 + M_s^2/4\mu$. At this order, μ_e is then unconstrained and can be set to zero. (As an aside, note that the Double-2SC phase is a $\tilde{Q}$-insulator. The simplest way to see this is to note that the CFL phase is a $\tilde{Q}$-insulator and that the two unpaired quarks and the one gapless quasiparticle in the Double-2SC phase are all $\tilde{Q}$-neutral.)

To order M_s^4, the free energy for electric and color neutral Double-2SC quark matter is

$$\Omega^{\rm neutral}_{\rm Double-2SC} = \Omega^{\rm neutral}_{\rm unpaired} + \frac{3M_s^4 - 32\Delta^2\mu^2}{16\pi^2}\ . \quad (3.26)$$

3.5 2SC vs. CFL Comparison

The first comparison to make is that between all the phases we have discussed (unpaired, CFL, 2SC, 2SCus, Double-2SC) upon making the assumption that M_s is the same in all phases and Δ is the same in all the paired phases. This comparison is shown in Fig. 1. We see that there is no value of Δ for which the Double-2SC phase is favored. In particular, for all values of Δ the CFL phase has lower free energy. We therefore disregard the Double-2SC phase henceforth.

If Fig. 1 were the end of the story, we would conclude that for $\Delta < M_s^2/4\mu$ unpaired quark matter is favored, whereas for $\Delta > M_s^2/4\mu$ CFL quark matter is favored. Precisely at $\Delta = M_s^2/4\mu$, unpaired, CFL, 2SC and 2SCus quark matter all have the same free energy. There is therefore (just barely) no 2SC window. Given the fragility of this conclusion, let us enumerate the effects we know we have left out:

- Effects which are of order M_s^6 and higher. We have discussed some of these above. We do not know how they all add up, but we can consistently neglect them.

- As mentioned above, we have neglected the possibility of kaon condensation in CFL quark matter. This can only lower the CFL free energy. It therefore works against opening a 2SC window.

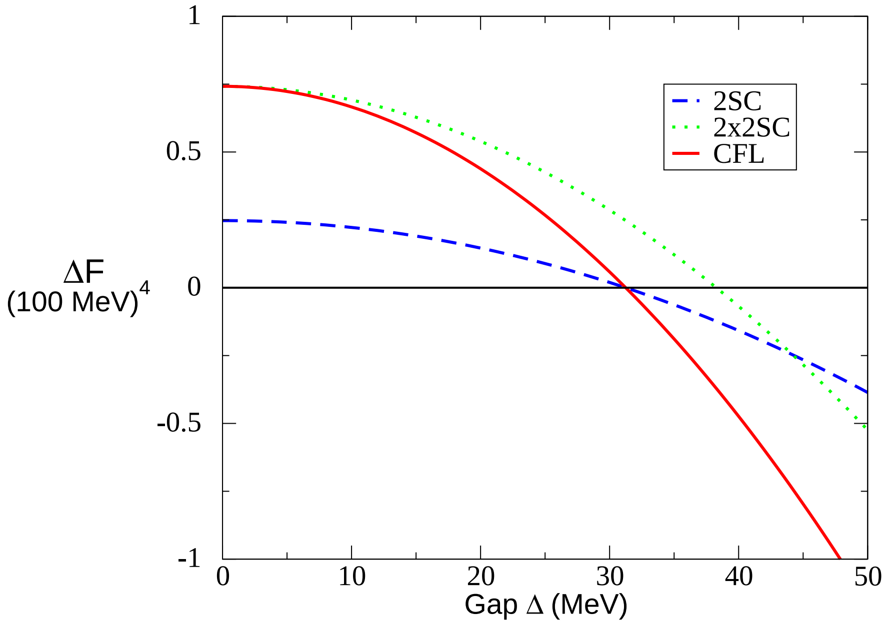

Figure 1: The free energy difference ΔF between various phases and unpaired quark matter, as a function of the gap Δ. The unpaired phase is therefore the $\Delta F = 0$ horizontal line. We show the CFL phase (red/solid), 2SC or 2SCus phase (blue/dashed), and Double-2SC phase (green/dotted). Below about 32 MeV the unpaired phase is favored, above it CFL is favored. There is no 2SC window. We have taken $\mu = 500$ MeV and $M_s = 250$ MeV.

- For a given interaction, the gap Δ is *not* the same in the 2SC and CFL phases. If the interaction is single-gluon exchange as at asymptotically high densities, the CFL gap is smaller than the 2SC gap by a factor of $2^{-1/3} \approx 0.79$ [20]. If the interaction is a point-like interaction with the quantum numbers of single-gluon exchange, the ratio of the CFL gap to the 2SC gap is ≈ 0.75 [13, 15]. The fact that the 2SC gap is somewhat bigger than the CFL gap tends to open up a 2SC window. This effect is comparable in magnitude (and opposite in sign) to that of kaon condensation.

- The biggest effect comes from the fact that M_s is not the same in the CFL and 2SC phases. The free energy of all the phases we have discussed includes a common $3/(4\pi^2)\ M_s^2\mu^2$ term, which plays no role in the comparison of Fig. 1. However when comparing phases with different M_s it will parametrically dominate the M_s^4 and Δ^2 effects on which we have focussed to this point. We now discuss this effect in more detail.

A full treatment of how the constituent mass M_s and gaps Δ vary between

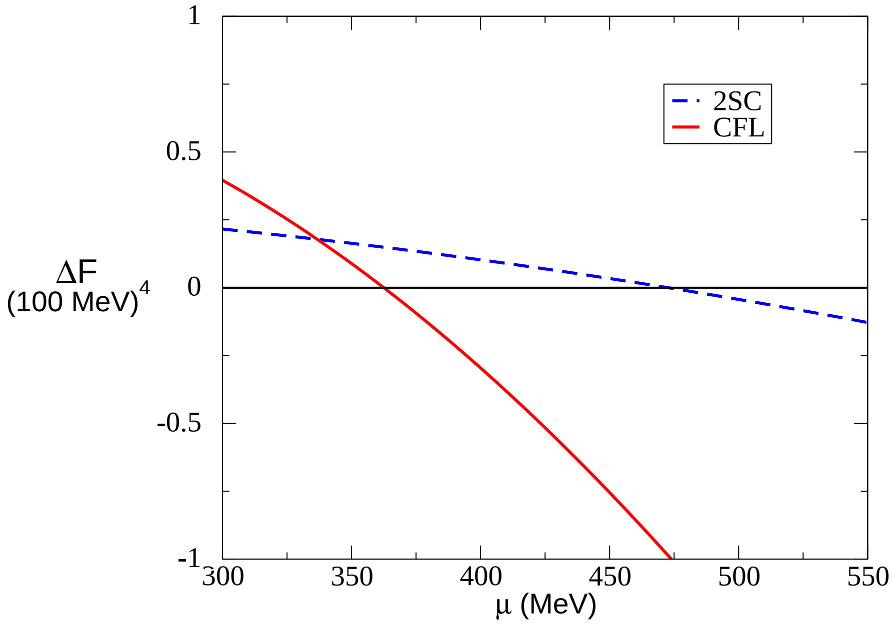

Figure 2: The free energy of the 2SC (blue/dashed) and CFL (red/solid) phases as a function of the quark chemical potential μ, relative to the unpaired phase with $M_s = 275$ MeV (horizontal black line). We have taken $M_s = 250$ MeV and $\Delta = 30$ MeV in the CFL phase, and $M_s = 275$ MeV and $\Delta = 40$ MeV in the 2SC phase. These parameters are completely ad hoc, but the resulting curves demonstrate quite clearly that the effect of the difference between Δ's, which seeks to open a 2SC window, is easily overpowered by the effect of even a fairly small difference between M_s's, which slams the window shut.

different phases would require analysis of the coupled gap equations in some model Hamiltonian, an exercise that goes beyond the general considerations we employ in this paper. Such an analysis has recently been done for several NJL models by Buballa and Oertel [21], albeit without enforcing electric and color neutrality. They find that M_s is considerably smaller in the CFL phase than in the 2SC phase. They do not consider the 2SCus phase, but it is reasonable to assume that M_s in this phase will be comparable to that in the 2SC phase.

In Fig. 2, we illustrate the effect of a relatively small difference in the value of M_s between phases. We plot the relative free energy of the CFL, 2SC, and unpaired phases, taking ($M_s = 250$ MeV, $\Delta = 30$ MeV) in the CFL phase and ($M_s = 275$ MeV, $\Delta = 40$ MeV) in the unpaired and 2SC phases. (In the models of Ref. [21], the change in M_s between the CFL and 2SC phases is significantly larger, of order 30%, and there is no analogue of our unpaired quark matter.) We see in Fig. 2 that the fragility of the conclusion of Fig. 1 is illusory: the 2SC phase is firmly excluded if M_s varies as Buballa and Oertel find. Because it is an order M_s^2 effect, the greatest uncertainty in our conclusions arises from our uncertainty in how much M_s changes between different phases. Our analysis has also neglected μ-independent contributions to the free energy that arise from the binding energy of the $\langle \bar{s}s \rangle$ condensate, and so favor the 2SC phase where M_s is larger. These are subleading relative to the $\mu^2 M_s^2$ term.

4 Conclusions

We find that, taking into account the constraints imposed by color neutrality, electrical neutrality, and weak equilibrium, the two-flavor color superconducting (2SC) phase is not expected to be found in compact stars. Our argument is model independent, and is based on an expansion in M_s/μ and Δ/μ. Since it is possible that in lower density quark matter M_s is not much smaller than μ, it would be of considerable interest to perform the full calculation for a specific model, solving coupled gap equations under the constraint of electrical and color neutrality, to see whether the 2SC phase vanishes as expected. An NJL model such as that of Buballa and Oertel [21] would be a natural starting point.

Our results indicate that two qualitative possibilities remain viable for the phase diagram of quark matter that is electrically and color neutral. The first is a single phase transition directly from hadronic matter to color-flavor locked quark matter. The second possibility is that there may be a window of intermediate densities in which we find quark matter which is, at the level of the analysis of this paper, unpaired. That is, as a function of increasing μ we first go from hadronic matter to unpaired quark matter and only at a higher μ make the transition to CFL quark matter.

If there is a window of "unpaired" three-flavor quark matter, there will certainly be pairing in this window: all that we have shown is that there will be no BCS pairing between quarks of different flavors. One possible pattern of pairing is the formation of $\langle uu \rangle$, $\langle dd \rangle$ and $\langle ss \rangle$ condensates. These must be either $J = 1$ or symmetric in color, and are therefore much smaller than the $J = 0$ color-antisymmetric condensates we have investigated in this paper. The gaps in these phases may be as large as of order 1 MeV [22], or could be much smaller [3]. In all the condensates we have discussed to this point, the Cooper pairs are made of quarks with equal and opposite momenta. Another possibility for pairing in the "unpaired" quark matter is crystalline color superconductivity [17, 23, 24, 25], which involves pairing between quarks whose momenta do not add to zero, as first considered in a condensed matter physics context in Refs. [26]. The unpaired quark matter of (3.6) is susceptible to the formation of a crystalline color superconducting condensate constructed from pairs of quarks with different flavors, both of which have momenta near their respective unpaired Fermi surfaces. Our work demonstrates that the electrically neutral unpaired quark matter of Section 3.1 is the correct starting point for an analysis of crystalline color superconductivity. Crystalline color superconductivity need not coexist with the 2SC phase, as previously thought.

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